10.3 IF ESTIMATION FOR MULTICOMPONENT SIGNALS [0]

10.3.1 Time-Frequency Peak IF Estimation

There is a wide range of applications where we encounter signals comprised of M components with different IF laws $f_m(t)$ and different envelopes $a_m(t)$, in additive noise. It is often desired from such an observed signal, to determine the number of components M, the IF law of each component and the corresponding envelope $a_m(t)$. This can be achieved by representing the observed signal $z(t)$ in a time-frequency (t-f) domain and use time-frequency filtering methods to recover the individual components [1]. Another approach involves extending algorithms for IF estimation of monocomponent FM signals to the case of multicomponent signals and design an algorithm that simultaneously tracks the various IF components of the observed signal [2,3]. Both approaches require the use of time-frequency distributions (TFDs) with very specific properties such as high time-frequency localization of the IF components and high reduction of cross-terms interferences.

The basic concept of instantaneous frequency is described in Pt. 1 of reference [4] and in Chapter 1 of this book. Methods of IF estimation are reported in Pt. 2 of [4] and in Chapter 10. Essential results are reproduced below, for greater clarity.

10.3.1.1 Spectrogram Peak IF estimation

Various approaches for IF estimation of monocomponent signals exist [4]. Most of these algorithms are suited to a particular class of signals, and both fixed and adaptive algorithms have been proposed. Our aim here is to approach the problem from a general viewpoint in order to define a general IF methodology that would be suitable for the largest class of signals found in practical applications. To illustrate this approach, we thus consider from the outset multicomponent signals in additive noise, which can be expressed as follows:

$$z(t) = \sum_{m=1}^{M} z_m(t) + \epsilon(t) = \sum_{m=1}^{M} a_m(t) e^{j\phi_m(t)} + \epsilon(t) \tag{10.3.1}$$

where the amplitudes $a_m(t)$ are the component amplitudes, $\phi_m(t)$ are the component phases, and $\epsilon(t)$ is a complex-valued white Gaussian noise process of independent and identically distributed (i.i.d.) real and imaginary parts with total variance σ_ϵ^2. The individual IF laws for each component are given by [1]:

$$f_m(t) = \frac{1}{2\pi} \frac{d\phi_m(t)}{dt} \quad ; \; m = 1, ..., M. \tag{10.3.2}$$

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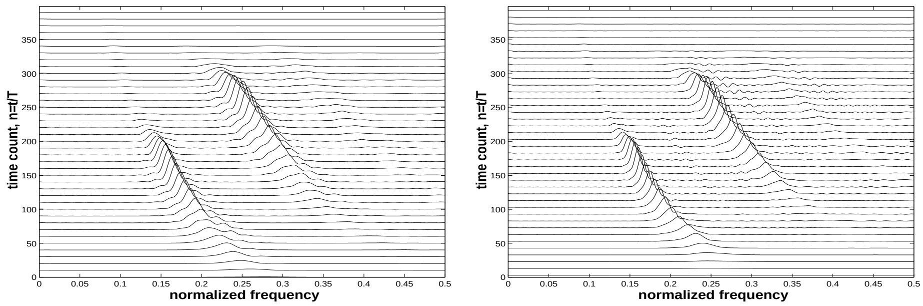

Fig. 10.3.1: Left: The spectrogram of a bat signal using a small analysis window. Right: The modified B-distribution of the same signal with parameter $\beta = 0.05$. Total signal length is $N = 400$ and sampling interval $\Delta t = 1$. The spectrogram cannot show the weakest component.

A conventional approach to represent and analyze such signals for IF estimation is to take the spectrogram of $z(t)$ and search for the peaks in the t-f domain (see Article 10.1). Curves formed by a continuum of these peaks describe the IF laws of the individual components of the observed signal $z(t)$, as illustrated in Fig. 10.3.1 using a bat signal. Analytically, this can be expressed as follows:

$$\hat{f}_m(t) = \arg[\max_f \rho_m(t, f)]; \quad 0 \leq f \leq f_s/2 \tag{10.3.3}$$

where $\rho_m(t, f)$ is the m^{th} peak of the spectrogram.

This spectrogram based approach has several advantages: it is easy to understand, easy to use, and there are no cross-terms producing unwanted interferences. A major disadvantage though is that the time-frequency resolution of the spectrogram for closely spaced components is often poor, especially if one of the components is much weaker, as illustrated in Fig. 10.3.2 for a two-component linear FM signal with one component weaker than the other. Fig. 10.3.2 also compares the performance of the spectrogram with the modified B-distribution (MBD) that was designed specifically for multicomponent IF estimation, as discussed later.

10.3.1.2 Peaks of WVD, PWVD, and RIDs

To improve upon the resolution of the spectrogram, various TFDs were proposed for IF estimation, one of the most important being the Wigner-Ville distribution (WVD). IF estimation using the peak of the Wigner-Ville distribution (WVD) is optimal for linear FM signals with high to moderate signal-to-noise ratios (SNRs) [4], but its performance degrades significantly at low SNRs, and in this case the cross WVD (XWVD) peak can be used as an IF estimator [5]. For polynomial FM signals it was shown that the polynomial WVD (PWVD) gives the best performance, especially at high SNRs (see [6] and Article 10.4). However, both WVD and PWVD suffer from cross-terms when used to analyze multicomponent signals. These cross-terms generate artifacts that obscure the (t-f) representation of the signal, leading to

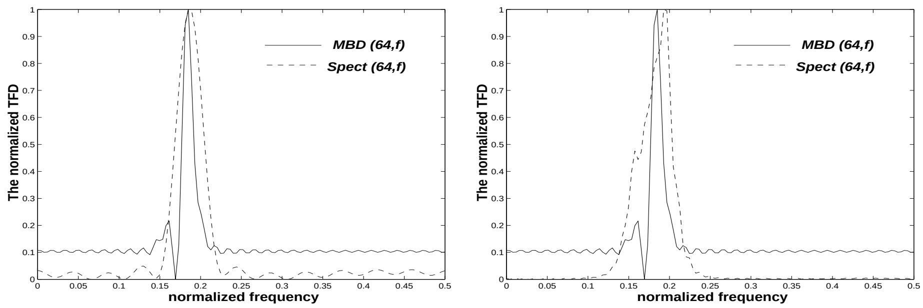

Fig. 10.3.2: Performance comparison between the spectrogram $\mathrm{Spect}(t,f)$ and the modified B-distribution (MBD) for $\beta = 0.06$ using a two-component noise-free linear FM signal at the sampling instant $n = t/\Delta t = 64$. Total signal length is $N = 128$ and the sampling interval is $\Delta t = 1$. The right component is five times larger in amplitude than the left component. Left: Spectrogram with small analysis window length ($\Delta = 23$). Right: Spectrogram with large analysis window length ($\Delta = 83$). In both cases the spectrogram fails to resolve the two components. In addition, time resolution is bad for a large window length.

the development of reduced interference distributions (RIDs) to remedy the problem [7]. Straightforward IF estimation using the peak of RIDs give an IF estimate that is biased from the true IF law, and this bias is different for different RIDs. Although reduced, cross-terms still exist and can obscure weak components, hence the need to define special purpose RIDs with efficient cross-terms reduction, high time-frequency resolution and minimum bias from the true IF laws, such as the MBD [3, 8].

10.3.2 Properties of IF Estimates Based on Quadratic TFDs

10.3.2.1 IF Estimates and Window Length

We consider an analytic signal $z(t)$ of the form $z(t) = ae^{j\phi(t)} + \epsilon(t)$ where the amplitude a is constant, and $\epsilon(t)$ is a complex-valued white Gaussian noise with independent identically distributed (i.i.d.) real and imaginary parts with total variance σ_ϵ^2. The IF of $z(t)$ is given by eq. (10.3.2), and it is assumed to be an arbitrary, smooth and differentiable function of time with bounded derivatives of all orders. The general equation for quadratic time-frequency representation of the signal $z(t)$ is given by [1]

$$\rho_z(t,f) = \mathop{\mathcal{F}}_{\tau\to f}[G(t,\tau) \mathop{*}_{(t)} K_z(t,\tau)]$$

where $G(t,\tau)$ is the time-lag kernel, $K_z(t,\tau) = z(t+\frac{\tau}{2})z^*(t-\frac{\tau}{2})$ is the signal kernel or the instantaneous autocorrelation function (IAF), and $\mathop{*}_{(t)}$ denotes time convolution. For smoothing and localization on the IAF we apply a window function $w_h(\tau) = \frac{\Delta t}{h}w(\frac{\tau}{2h})$ on the instantaneous autocorrelation $K_z(t,\tau)$, where $w(t)$ is a real-valued symmetric window with unity length, i.e., $w(t) = 0$ for $\mid t \mid > \frac{1}{2}$; hence the window length is h.

The TFD is now dependent on the window length h as follows:

$$\rho_{z,h}(t,f) = \underset{\tau\to f}{\mathcal{F}}[w_h(\tau)G(t,\tau) \underset{(t)}{*} K_z(t,\tau)]. \tag{10.3.4}$$

If $\rho_{z,h}(t,f)$ is discretized over time, lag, and frequency then we have

$$\rho_{z,h}(n,k) = \sum_{l=-N_s}^{N_s-1} \sum_{m=-N_s}^{N_s-1} w_h(m\Delta t) K_z(l\Delta t, 2m\Delta t) G(n\Delta t - l\Delta t, 2m\Delta t) e^{-j2\pi \frac{km}{2N_s}} \tag{10.3.5}$$

where $2N_s$ is the number of samples and Δt is the sampling interval.

The IF estimate is a solution of the following optimization

$$\hat{f}_h(t) = \arg[\max_f \rho_{z,h}(t,f)] \quad ; \; 0 \leq f \leq f_s/2 \tag{10.3.6}$$

where $f_s = 1/\Delta t$ is the sampling frequency.

10.3.2.2 Bias and Variance of the IF Estimate

By extending the results in [9], the estimation bias and variance are found to be [3]

$$E[\Delta\hat{f}_h(t)] = \frac{L_h(t)}{2F_h}, \quad \mathrm{var}(\Delta\hat{f}_h(t)) = \frac{\sigma_\epsilon^2}{2\mid a\mid^2}[1 + \frac{\sigma_\epsilon^2}{2\mid a\mid^2}]\frac{E_h}{F_h^2} \tag{10.3.7}$$

where

$$\begin{aligned} \Delta\hat{f}_h(t) = \tfrac{1}{2\pi}\phi'(t) - \hat{f}(t) \; ; \quad F_h &= \int_{-\infty}^{\infty} \sum_{m=-\infty}^{\infty} w_h(m\Delta t)(2\pi m\Delta t)^2 G(u, 2m\Delta t)du \\ L_h(t) &= \int_{-\infty}^{\infty} \sum_{m=-\infty}^{\infty} w_h(m\Delta t)\Delta\phi(u, m\Delta t)(2\pi m\Delta t)G(t-u, 2m\Delta t)du \\ E_h &= \int_{-\infty}^{\infty} \sum_{m=-\infty}^{\infty} w_h(m\Delta t)^2(2\pi m\Delta t)^2 G(u, 2m\Delta t)du \end{aligned} \tag{10.3.8}$$

where $\Delta\phi(t,\tau) = \phi(t+\tau/2) - \phi(t-\tau/2) - \tau\phi^{'}(t)$.

Equations (10.3.7) and (10.3.8) indicate that the bias and the variance of the estimate depend on the lag window length h for any kernel $G(t,\tau)$. To see how the bias and the variance vary with h, asymptotic analysis as $\Delta t \to 0$ is necessary for the chosen TFD.

10.3.2.3 TFD Properties Needed for Multicomponent IF Estimation

The results above indicate that a general method for IF estimation of multicomponent FM signals in additive Gaussian noise that is based on quadratic time-frequency distributions requires the quadratic TFDs to satisfy the following conditions [3]:

(1) $\rho(t, f)$ should have a high time-frequency resolution while suppressing cross-terms efficiently so as to give a robust IF estimate for mono- and multicomponent FM signals.
(2) $\rho(t, f)$ should enable amplitude estimation for the individual components of the signal, as the amplitude is necessary for evaluating the variance of the IF estimate for each component [2, 3, 9], and to allow for the reconstruction of the individual components of the signal.
(3) the choice of the lag window length should lead to a bias-variance tradeoff (see eqs. (10.3.13) and (10.3.14)).

Although some TFDs, like Choi-Williams distribution (CWD) and the spectrogram, can satisfy some of these conditions, they do not meet the second requirement, i.e. allowing direct amplitude estimation. The design of TFDs which satisfies all of these required properties is considered next.

10.3.3 Design of Quadratic TFDs for Multicomponent IF Estimation

10.3.3.1 Desirable Time-Lag Kernel

A TFD referred to as the B-distribution (BD) was proposed and shown to be superior to other fixed-kernel TFDs in terms of cross-terms reduction and resolution enhancement [10]. As it does not allow direct component amplitudes estimation [2], as per the second condition on TFDs required for multicomponent IF estimation listed above, the BD kernel was modified as [3]

$$G(t, \tau) = \mathrm{G}_\beta(t) = k_\beta / \cosh^{2\beta}(t) \tag{10.3.9}$$

where β is a real positive number and $k_\beta = \Gamma(2\beta)/(2^{2\beta-1}\ \Gamma^2(\beta))$, Γ stands for the gamma function. This modified B-distribution $\mathrm{MBD}(t, f)$ is also referred to as the hyperbolic T-distribution (HTD) in [8].

10.3.3.2 Relevant Properties of the Modified B-Distribution (MBD)

Most of the desirable properties of time-frequency distributions relevant to IF estimation (as explained in [1] and [4]) are satisfied by the MBD kernel. In particular, realness, time-shift and frequency shift invariance, frequency marginal and group delay, and the frequency support properties are satisfied. The time support property is not strictly satisfied, but it is approximately true [3]. The three required conditions listed in Subsection 10.3.2.3 are discussed in detail below.

(1) Reduced interference and resolution: This property is satisfied by MBD. For example, consider the sum of two complex sinusoidal signals $z(t) = z_1(t) + z_2(t) = a_1 e^{j(2\pi f_1 t + \theta_1)} + a_2 e^{j(2\pi f_2 t + \theta_2)}$ where a_1, a_2, θ_1 and θ_2 are constants. The TFD of the signal $z(t)$ is obtained as [3]

$$\mathrm{MBD}(t, f) = a_1^2\ \delta(f - f_1) + a_2^2\ \delta(f - f_2) + 2a_1 a_2 \gamma_\beta(t)\ \delta[f - (f_1 + f_2)/2] \tag{10.3.10}$$

where $\gamma_\beta(t) = |\Gamma(\beta + j\pi(f_1 - f_2))|^2 \cos\Big(2\pi(f_1 - f_2)t + \theta_1 - \theta_2\Big)/\Gamma^2(\beta)$. The cross-terms are oscillatory in time and depend on the frequency separation between signal components. If f_1 and f_2 are well separated then the term $|\Gamma(\beta + j\pi(f_1 - f_2))|^2$ can be substantially reduced, while $\Gamma^2(\beta)$ can be made high if β is small. When f_1 and f_2 are not well separated, the MBD still performs better than most quadratic TFDs (see Article 7.4).

(2) Direct amplitude and IF estimation: The MBD allows direct IF estimation by peak localization, i.e., at any time instant t, it has an absolute maximum at $f = \frac{1}{2\pi}\frac{d\phi(t)}{dt}$ for linear FM signals . For non-linear FM signals this estimate is biased, but this bias can be accounted for in the adaptive IF estimation, as presented next.

For an FM signal of the form $z(t) = a\ e^{j\phi(t)}$, the MBD is approximated by [3]

$$\mathrm{MBD}(t,f) \approx |\ a\ |^2 \int_{-\infty}^{\infty} \mathrm{G}_\beta(t-u)\ \delta\ [\frac{1}{2\pi}\phi^{'}(u) - f]du = |a|^2 \mathrm{G}_\beta(t - \psi(f))\psi^{'}(f) \tag{10.3.11}$$

where ψ is the inverse of $\frac{1}{2\pi}\phi^{'}$, i.e., $\frac{1}{2\pi}\phi^{'}(\psi(f)) = f$. Assuming that $\psi^{'}(f)$ is not a highly peaked function of f and knowing that $\mathrm{G}_\beta(t - \psi(f))$ is peaked at $t = \psi(f)$, the absolute maximum of $\mathrm{MBD}(t,f)$ for any time t would be at $\psi(f) = t$, or $f = \frac{1}{2\pi}\phi^{'}(t)$, which is the IF of the FM signal $z(t)$. For non-linear FM signals, the energy peak of the MBD is actually biased from the IF because of the extra term $\sum_{k=3(k\ \mathrm{odd})}^{\infty} \frac{\tau^{k-1}}{k!2^{k-1}}\phi^{(k)}(u)$. The major contribution in this term is due to $\phi^{(3)}(u)$. Therefore at the instants of rapid change in the IF law the bias is not negligible and eq. (10.3.11) would not be an accurate approximation to the MBD unless suitable windowing in the lag direction is used.

For linear FM signals we have $\phi^{(k)}(t) = 0$ for $k \geq 3$. Assuming that $z(t) = ae^{j2\pi(f_o t + \frac{\beta_o}{2}t^2)}$, where f_o and β_o are constants, we have

$$\mathrm{MBD}(t,f) = \frac{1}{\beta_o}|a|^2 \mathrm{G}_\beta\Big(t - \frac{1}{\beta_o}(f - f_o)\Big) \tag{10.3.12}$$

which has an absolute maximum at $f = f_o + \beta_o t$, the IF of the linear FM signal $z(t)$. As $\beta_o \to 0$, i.e., $z(t)$ approaches a sinusoid, we have $\mathrm{MBD}(t,f) \to |a|^2\delta(f - f_o)$, in accordance with eq. (10.3.10).

As for amplitude estimation, eqs. (10.3.11) and (10.3.12) indicate that the MBD can support amplitude estimation [3].

(3) Asymptotic Formulas Using MBD: The asymptotic formulas for the variance and the bias as $\Delta t \to 0$ using a rectangular lag window are given by [3]:

$$\mathrm{var}(\Delta\hat{f}_h(t)) = \frac{3\sigma_\epsilon^2}{2\pi^2\ |\ a\ |^2}[1 + \frac{\sigma_\epsilon^2}{2|a|^2}]\frac{\Delta t}{h^3} \tag{10.3.13}$$

and

$$E(\Delta\hat{f}_h(t)) = \frac{h^2}{80}\int_{-\infty}^{\infty}\frac{\lambda(u)du}{\cosh^{2\beta}(t-u)}\ ; \quad E(\Delta\hat{f}_h(t)) \le \frac{M_2}{40}h^2 \tag{10.3.14}$$

where $\lambda(t) = f^{(2)}(t+\tau_1) + f^{(2)}(t-\tau_1)$, $f^{(2)}(t)$ is the second derivative of the IF, and $\sup_t|f^{(2)}(t)| \le M_2$. For small h, the optimal window length that minimizes the mean squared error is obtained by extending the result in [6] as:

$$\mathrm{h}_{\mathrm{opt}}(t) = \Big[\frac{1800\sigma_\epsilon^2\Delta t(1+\frac{\sigma_\epsilon^2}{2|a|^2})}{\pi^2|a|^2(f^{(2)}(t)\underset{(t)}{*}1/\cosh^{2\beta}(t))^2}\Big]^{\frac{1}{7}}. \tag{10.3.15}$$

Thus, the optimal window length depends on the second derivative of the instantaneous frequency $f^{(2)}(t)$, which is time and signal dependent. Eqs. (10.3.13) and (10.3.14) indicate that the variance and bias of the IF estimate using MBD have the same rates of change with respect to the window length h as those using WVD [9].

10.3.3.3 Examples of Quadratic TFDs Suitable for Multicomponent IF Estimation

TFDs with time-only (or lag-independent) kernels constitute a subclass of the quadratic class of TFDs. These TFDs share the important properties of cross-terms suppression, high-resolution, and supporting amplitude estimation, making them well suited for multicomponent IF estimation. The modified B-distribution (a. k. a. hyperbolic T-distribution) was defined earlier in this section. Another example is the exponential T-distribution, which is defined in terms of its time-lag kernel as [8]

$$G(t,\tau) = \mathrm{G}_\eta(t) = \sqrt{\eta/\pi}\exp(-\eta t^2)$$

where η is a real parameter and $\sqrt{\eta/\pi}$ is a normalization factor. The resulting TFD used for multicomponent IF estimation is then given by eq. (10.3.4).

10.3.4 An Adaptive Algorithm for Multicomponent IF Estimation

Eq. (10.3.15) shows that the optimal window length using the MBD is a function of time and depends on the second derivative of the IF law $f^{(2)}(t)$; it decreases when the IF law $f(t)$ has a high variation. Hence a time-varying window length is needed to optimize the estimation. The Stanković-Katkovnik adaptive algorithm developed in [9] for monocomponent FM signals can be used since the IF estimation variance is a continuously decreasing function of h while its bias is continuously increasing, as shown in eqs. (10.3.13) and (10.3.14); see also Article 10.2. These conditions are necessary for bias-variance tradeoff such that the algorithm converges at the optimum window length that resolves this tradeoff. It is shown in [9] that, if h is

small enough then the IF estimate $\hat{f}_h(t)$ is inside the confidence interval D defined as follows

$$D = [\hat{f}_h(t) - 2\kappa\sqrt{\mathrm{var}(\Delta\hat{f}_h(t))}, \hat{f}_h(t) + 2\kappa\sqrt{\mathrm{var}(\Delta\hat{f}_h(t))}] \tag{10.3.16}$$

with Gaussian probability $P(\kappa)$, κ being a parameter (usually 2); while for large h, $\hat{f}_h(t)$ is outside D. Hence, if we consider an increasing sequence of window lengths $\{h_r | r = 1 : N\}$ (N being the number of samples) and calculate the MBD (and hence $\hat{f}_{h_r}(t)$) for each h_r, then all $\{D_r\}$ have at least one point in common (which is $\hat{f}_{h_r}(t)$) if h_r is sufficiently small. The first h_r for which D_{r-1} and D_r have no point in common is considered optimal as it decides the bias-variance tradeoff.

The estimates for the amplitude of the signal a and the variance of noise σ_ϵ^2 [used in eq. (10.3.13) and implicitly in eq. (10.3.16)] were given in [9] as:

$$\widehat{a}^2 + \widehat{\sigma}_\epsilon^2 = \frac{1}{N}\sum_{n=1}^{N} \mid z(n\Delta t) \mid^2 \ ; \quad \widehat{\sigma}_\epsilon^2 = \frac{1}{2N}\sum_{n=2}^{N} \mid z(n\Delta t) - z((n-1)\Delta t) \mid^2 \tag{10.3.17}$$

where N is the number of samples. For further details of this adaptive algorithm see [9].

For a multicomponent analytic signal of the form stated in eqs. (10.3.1) and (10.3.2), with $\{a_m\}$ constant, we can use the extension of the monocomponent IF estimation algorithm in [9] for multicomponent signals as described in [2, 3]. This algorithm tracks component maxima in the time-frequency plane and requires a threshold $\mathcal{T}_\rho(t)$ so as to ignore the local maxima caused by the cross-terms and windowing. In fact, $\mathcal{T}_\rho(t)$ is application and distribution dependent.

The algorithm requires the knowledge of the confidence intervals $D_{r,m}$ for each component, where r refers to the window length (h_r) and m refers to the signal component. The calculation of $D_{r,m}$ depends on the estimation of the individual amplitudes a_m of the components. Using the MBD, the actual amplitudes $|\hat{a}_m|$ can be estimated as shown in [3]. Using $|\hat{a}_m|^2$ and $\hat{\sigma}^2$ to calculate $\mathrm{var}(\Delta\hat{f}_h(t))$ [given by eq. (10.3.13) for $\mathrm{MBD}(t, f)$], we can define the confidence intervals $\{D_{r,m}\}$ for all components as in [2, 3]. The IF $f_m(t)$ is contained in at least one of the confidence intervals $\{D_{r,m}\}$ if h_r is sufficiently small, and the optimal window length is the first h_r (from the increasing sequence $\{h_r | r = 1 : N\}$) for which $D_{r-1,m}$ and $D_{r,m}$ have no point in common.

Example: We consider a three-component FM signal $z(n\Delta t)$ with amplitudes $a_1 = 0.5$, $a_2 = 1$, and $a_3 = 1.5$ and non-linear IF laws: $f_1 = 47 + 2.5\sinh^{-1}(20(n\Delta t - 0.2))$, $f_2 = 30 + 2.5\mathrm{sgn}(40(n\Delta t - 0.6))$, and $f_3 = 10 + 2\sin(10(n\Delta t - 0.7))$, with $SNR = 15$ dB, $\beta = 0.1$, $\kappa = 2$, $0 \le n\Delta t \le 1$, and $\Delta t = 1/128$. Fig. 10.3.3 shows the result of the tracking adaptive algorithm for IF estimation of $z(n\Delta t)$ using the peaks of the MBD and the spectrogram.

Fig. 10.3.4 shows the conventional peak IF estimation for the same signal using MBD and the spectrogram. Both TFDs fail to give a robust IF estimation at the

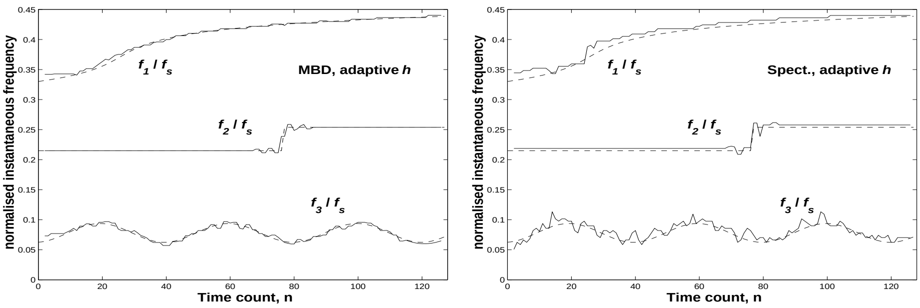

Fig. 10.3.3: Left: Adaptive IF estimation using the peak of the MBD for a three-component FM signal with total length $N = 128$, $SNR = 15$ dB, and $\Delta t = 1/128$. Dashed lines represent the true IF laws. Right: Adaptive IF estimation using the peak of the spectrogram for the same signal, assuming that component amplitudes are known.

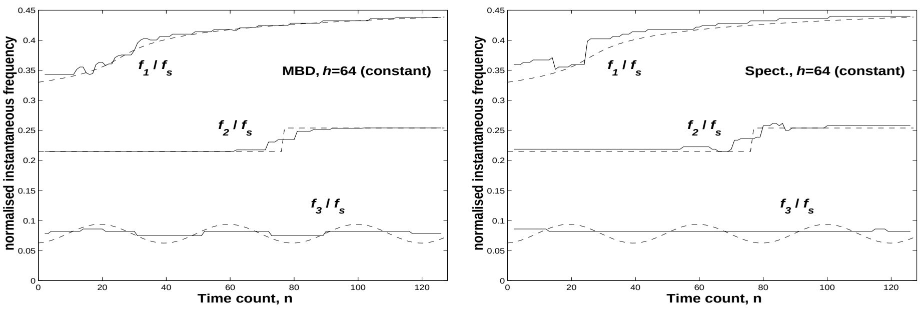

Fig. 10.3.4: Left: IF estimation for the three-component FM signal as in Fig. 10.3.3 using the conventional (constant window) peak estimation. Left: MBD peak IF estimation. Right: Spectrogram peak IF estimation. (In addition to failure of IF estimation at instants of rapid frequency change, the spectrogram has poor tracking performance. Note also that both methods cannot track the continuously varying frequency of the third component.)

instants of rapid frequency change. In addition, the spectrogram has poor time-frequency resolution, both in adaptive and constant-window IF estimation.

10.3.5 Summary and Conclusions

Concurrent IF estimation of the separate components of a multicomponent FM signal using TFD peaks location requires conditions on the selection of a suitable quadratic TFD. Required properties are: (1) high time-frequency resolution while suppressing cross-terms efficiently, (2) the TFD to enable direct amplitude estimation for the individual components, (3) the variance of the IF estimate using the TFD should be a continuously decreasing function of the lag window length while the bias is continuously increasing. Quadratic time-frequency distributions that satisfy these conditions were presented and discussed. A constant-window tracking

algorithm may not give a robust IF estimate if the IF changes rapidly with time due to the effect of the higher-order derivatives of the IF law. Hence an adaptive algorithm is used for robust multicomponent IF estimation.

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